MUON COLLIDER PHYSICS

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OUTLINE

The Machine

- Parameters a theorist needs to know.
- Some $+$'s and $-$'s from a physics point of view.

The Physics

- Front end physics.
- Z physics.
- Higgs physics.
- Precision m_W and m_t measurements.
- Leptoquarks
- R-parity violation in supersymmetry.
- Strong WW sector physics.
- New Z'
- Standard supersymmetry.

Will mention advantages, disadvantages and complementarity relative to other colliders.

Will emphasize unique physics opportunities for a first, low-energy muon collider.

WHAT WE NEED TO KNOW ABOUT THE MACHINE

A muon collider (MC) facility can be developed in stages, each successive stage building upon the previous one. Three stages are currently envisioned.

- Low-energy Higgs factory collider: $\sqrt{s} \sim 100$ GeV.
- Intermediate-energy collider: $\sqrt{s} \lesssim 500$ GeV.
- High-energy collider: $\sqrt{s} \sim 3-4$ TeV.

The instantaneous luminosity, $\mathcal{L}$, that can be achieved at each stage is still somewhat uncertain. For rather conservative designs of relatively low cost, current minimal expectations are:

- $\mathcal{L} \sim 1, 2, 10 \times 10^{31}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ at $\sqrt{s} \sim 100$ GeV for beam energy resolutions of $R = 0.003\%, 0.01\%, 0.1\%$, respectively;
- $\mathcal{L} \sim 1 \times 10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, at $\sqrt{s} \sim 400$ GeV for $R \sim 0.14\%$;
- $\mathcal{L} \sim 1 \times 10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, at $\sqrt{s} = 3-4$ TeV with $R \sim 0.16\%$.

(For yearly integrated luminosities, we use the standard convention of $\mathcal{L} = 1 \times 10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1} \Rightarrow L = 1\ \mathrm{fb}^{-1}/\mathrm{yr}$.) The above results can be partly understood from the approximate fixed muon number scaling law:

$$\mathcal{L} \propto E^{11/7} \times R^{4/7}.$$

Can a combination of money and clever ideas allow the ultimate $\mathcal{L}$ values to be larger?

Some +'s and −'s and Critical Requirements for the Muon Collider

First, there are some important advantages as compared to an electron collider.

- There is less bremsstrahlung and no beamstrahlung.
- Beam energy resolution can be substantially better — in particular, with beam compression techniques $R = 0.003\%$ can be achieved at the low-energy Higgs factory so that the Gaussian spread in $\sqrt{s}$, given by
$$\sigma_{\sqrt{s}} \sim 2\ \mathrm{MeV} \left(\frac{R}{0.003\%}\right) \left(\frac{\sqrt{s}}{100\ \mathrm{GeV}}\right) , \tag{0.1}$$
can be as small as the natural width of a light SM-like Higgs boson.
- The beam energy can be very precisely tuned: $\Delta E_{\rm beam} \sim 10^{-5} E_{\rm beam}$ is 'easy'; 10^{-6} is achievable and very important for scanning a narrow Higgs boson and precision m_W and m_t measurements. (To achieve such precision, power supplies stable at the 10^{-6} level are required and one must plan to monitor the beam energy continuously via spin rotation measurements.)
- There are no obstacles to upgrading the energy of a low-$\sqrt{s}$ muon collider to $\sqrt{s} = 3 - 4$ TeV. One simply increases the amount of recirculating acceleration and builds a (relatively cheap) storage ring designed for the desired energy.
- Multiple interaction regions in the final storage ring, allowing full luminosity for several detectors, might not be impossible.

Other positive features of the muon collider include the following. It can be built in stages. The proton driver, intense muon source, cooled low-energy muon beam, and so forth, that will sequentially become available as the machine is constructed would all have important uses of their own. The energy can be increased by additions that are modest in physical size and don't involve substantial new technology. Particularly noteworthy are the following points.

- If constructed at Fermilab, the ~ 50 GeV μ^+ and μ^- beams needed for the Higgs factory could be collided with the 1 TeV proton beam of the Tevatron, yielding a μp analogue of HERA with roughly $\sqrt{2}$ times as large center of mass energy and larger luminosity. Eventual higher energy, higher luminosity muon beams would result in a μp collider with physics reach vastly exceeding that of HERA.

- Since the cost of a final storage ring is modest, several would be built as the energy of the machine is increased, each designed to optimize luminosity at specific energies designed for specific physics goals (to be discussed in more detail later). An incomplete list is the following.

  – If a light ($m_h \lesssim 2m_W$) SM-like Higgs boson has been observed (*e.g.* at the LHC), the first energy goal and ring constructed would be for factory-like s-channel production and study at $\sqrt{s} \sim m_h$.

  – A second energy goal and ring would be for operation at high $\mathcal{L}$ near the Zh threshold. (This would actually be the first goal if a SM-like Higgs has been observed and has $m_h > 2m_W$.) One would choose $\sqrt{s}$ so that the Zh cross section is maximal, thereby allowing precise measurement of many Higgs boson properties.

(Even if $m_h < 2m_W$, there are important Higgs properties that are not easily measured in s-channel production.) A fairly precise determination of m_h from the $\sigma(Zh)$ threshold rise would also be possible.

– Exceptionally precise measurements of m_W and of m_t, α_s, Γ_t, are possible with rings that achieve full luminosity at $\sqrt{s} \sim 2m_W$ and/or $\sqrt{s} \sim 2m_t$, respectively. If no Higgs boson is seen at the LHC, then this would constitute an important first goal for the muon collider.

– Factories for s-channel production of any new particle with $\mu^+\mu^-$ couplings would be possible. Possibilities include a new Z' and a sneutrino with R-parity-violating coupling to $\mu^+\mu^-$.

Once the accelerator is operating at high energy, beams of different energy appropriate to the different rings could be extracted and the luminosity could be shared among the various rings (and with the μp collider). This would allow simultaneous pursuit of many different types of physics at different detectors, as possibly desirable from both a physics and a sociological point of view.

There are two clear disadvantages of a muon collider:

- A $\gamma\gamma$ collider is not possible at a muon collider facility.
- Some polarization is automatic, but large polarization implies sacrifice in luminosity at a muon collider. This is because large polarization is achieved by keeping only the larger p_z muons emerging from the target, rather than collecting nearly all the muons.

FRONT-END PHYSICS

- Physics with low-energy hadrons ($p, \overline{p}, K, \pi$).
- Neutrino physics (NUMI source, e.g.)
- Stopped muon physics ($g-2$, e.g.)

Conclusion: The front-end would provide opportunities in these areas that far exceed what is possible at currently available/planned facilities.

Z PHYSICS

The muon collider could be run as a Z factory that would quickly exceed statistical levels achieved at LEP and SLC/SLD.

- $\sigma(Z)_{\rm peak} \sim 6 \times 10^7$ fb.
- $\mathcal{L}_{\rm LEP} \sim 2.4 \times 10^{31}{\rm cm}^{-2}{\rm s}^{-1} \Rightarrow 1.5 \times 10^7$ Z's per year; no polarization.
- $\mathcal{L}_{\rm SLD} < 1 \times 10^{30}{\rm cm}^{-2}{\rm s}^{-1}$ (never achieved) $\Rightarrow < 6 \times 10^5$ Z's per year; high polarization for e^- beam.
- $\mathcal{L}_{\rm MC} \sim 1 \times 10^{32}{\rm cm}^{-2}{\rm s}^{-1}$ ($R = 0.1\%$ perfectly ok) $\Rightarrow \sim 6 \times 10^7$ Z's per year; partial ($\sim 30\%$?) polarization for *both* beams.
- Substantial polarization ($\gtrsim 50\%$) for both beams can be achieved at a MC if something like a factor of 4 sacrifice in luminosity is accepted $\Rightarrow \sim 1.5 \times 10^7$ Z's per year, a factor of > 25 better than SLD design.

Conclusion: If there are things that need to be better understood about the Z (CP violation, FCNC, precision A_{LR}) then a muon collider would be a good machine for such studies. (Of course, at NLC, *with modified interaction region for high* $\mathcal{L} \sim 10^{33}{\rm cm}^{-2}{\rm s}^{-1}$ *at low* $\sqrt{s} = m_Z$, can do even better.)

HIGGS PHYSICS:

- Away from s-channel Higgs pole, $\mu^+\mu^-$ and e^+e^- colliders have similar capabilities for same $\sqrt{s}$ and $\mathcal{L}$ (barring unexpected detector backgrounds at muon collider).

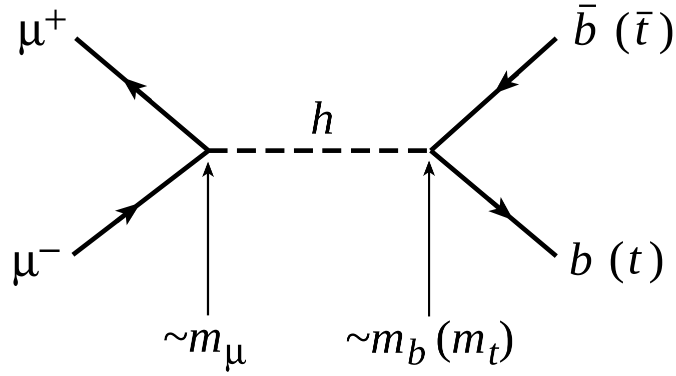

Figure 1: Feynman diagram for s-channel production of a Higgs boson.

- The totally unique feature of a muon collider is the possibility of s-channel Higgs production, $\mu^+\mu^- \to h$.

  Compute $\overline{\sigma}_h$ by convoluting a Gaussian $\sqrt{s}$ distribution of width $\sigma_{\sqrt{s}}$ with the standard s-channel Breit Wigner Higgs resonance cross section. For $\sqrt{s} = m_h$, one obtains

$$\overline{\sigma}_h \simeq \frac{\pi\sqrt{2\pi}\Gamma(h \to \mu\mu)\, BF(h \to X)}{m_h^2 \sigma_{\sqrt{s}}} \times \left(1 + \frac{\pi}{8}\left[\frac{\Gamma_h^{\rm tot}}{\sigma_{\sqrt{s}}}\right]^2\right)^{-1/2} . \quad (0.2)$$

  Eq. (0.2) $\Rightarrow$ small $\Gamma_h^{\rm tot}$ and $\sigma_{\sqrt{s}} \sim \Gamma_h^{\rm tot} \to$ big $\overline{\sigma}_h$.

  Although smaller R implies smaller $\mathcal{L}$, the correlation is such that for small $\Gamma_h^{\rm tot}$ (*e.g.* a SM-like Higgs boson with $m_h \lesssim 2m_W$) it is best to use the smallest R that can be achieved.

Standard Model-Like Higgs

$m_h \lesssim 2m_W$ is required for good s-channel cross section. Above that Γ_h becomes big and $\overline{\sigma}_h \propto BF(h \to \mu^+\mu^-)$ is too small.

Strategy: First center on $\sqrt{s} \sim m_h$ and then measure Higgs properties.

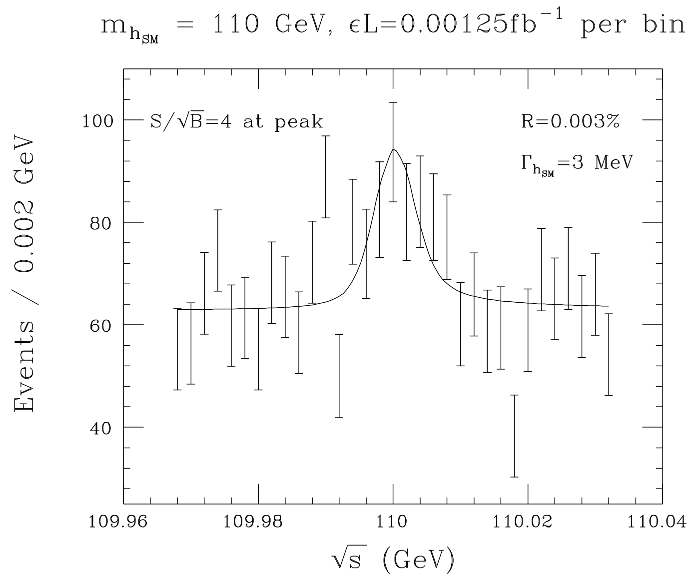

Figure 2: Initial scan for centering on $m_{h_{SM}} = 110$ GeV Higgs boson.

- For a SM-like Higgs with $m_h \lesssim 2m_W$: $\Delta m_h \sim 100$ MeV from LHC ($L = 300$ fb^{-1}); $\Delta m_h \sim 50$ MeV from $\sqrt{s} = 500$ GeV NLC ($L = 200$ fb^{-1}).

  $\Rightarrow$ can design a final ring for $\sqrt{s} \sim m_h$.

  Once operating, scan over the Δm_h interval so as to center on $\sqrt{s} \simeq m_h$ within a fraction of $\sigma_{\sqrt{s}}$.

  – "typical case" — $m_h \sim 110$ GeV, $\sigma_{\sqrt{s}} \sim 2$ MeV, $\Delta m_h \sim 100$ MeV $\Rightarrow \sim 50$ points needed to center within $\lesssim \sigma_{\sqrt{s}}$. Each point requires $L \sim 0.0015$ fb^{-1} to observe or eliminate the h at the 3σ level. $\Rightarrow$ $L = 0.075$ fb^{-1} needed to center $\Rightarrow$ for $L = 0.1$ fb^{-1}/yr, centering might take 1 yr.

– worst case — $m_h = m_Z$; a factor of 50 more $L_{\rm tot}$ needed $\Rightarrow$ 4 years even at $L = 1$ fb^{-1}/yr.

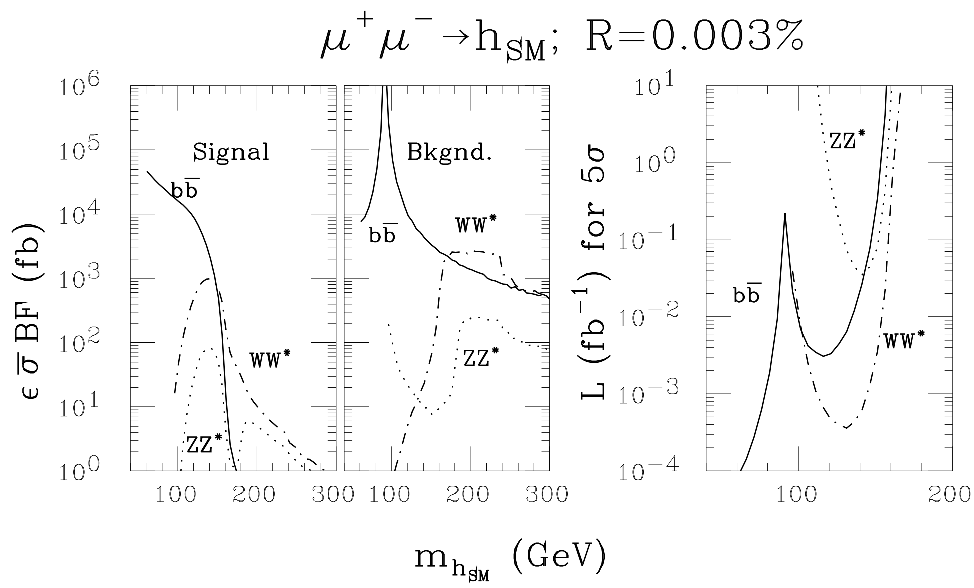

Figure 3: SM rates and L required for 5σ observation as a function of $m_{h_{SM}}$.

- The crucial measurements are then:
  - The very tiny Higgs width: $\Gamma_h^{\rm tot} = 1-10$ MeV for a SM-like Higgs with $m_h \lesssim 140$ GeV (i.e. mass as predicted for h^0 of SUSY).
  - $\sigma(\mu^+\mu^- \to h \to X)$ for $X = b\overline{b}, WW^\star, ZZ^\star$.*

Accuracy achievable? Employ optimized 3-point scan.

At $L = 0.4$ fb^{-1} the errors for $\sigma BF(h_{SM} \to b\overline{b})$ are still generally small but those for $\Gamma_{h_{SM}}^{\rm tot}$ are uncomfortably large. In fact, errors for $\Gamma_{h_{SM}}^{\rm tot}$ obtained indirectly using a combination of $L = 600$ fb^{-1} LHC data, $L = 200$ fb^{-1} NLC data, and $L = 50$ fb^{-1} $\gamma\gamma$-collider data are often better: $\sim 19\%$ for $m_{h_{SM}} \lesssim 120$ GeV and $\sim 10\% - 13\%$ for 130 GeV $\lesssim m_{h_{SM}} \lesssim 180$ GeV.

*Note from Eq. (0.2) that $\sigma(\mu^+\mu^- \to h \to X)$ provides a determination of $\Gamma(h \to \mu^+\mu^-)BF(h \to X)$ unless $\sigma_{\sqrt{s}} \ll \Gamma_h^{\rm tot}$.

Table 1: Percentage errors (1σ) for $\sigma BF(h_{SM} \to b\bar{b}, WW^\star, ZZ^\star)$ (extracted from channel rates) and $\Gamma^{\rm tot}_{h_{SM}}$ for s-channel Higgs production at the MC assuming beam energy resolution of $R = 0.003\%$. Results are presented for two integrated four-year luminosities: $L = 4~{\rm fb}^{-1}$ ($L = 0.4~{\rm fb}^{-1}$). An optimized three-point scan is employed [which, for the cross section measurements, is equivalent to $L \sim 2~{\rm fb}^{-1}$ ($L = 0.2~{\rm fb}^{-1}$) at the $\sqrt{s} = m_{h_{SM}}$ peak].

Quantity	Errors			
Mass (GeV)	**80**	m_Z	**100**	**110**
$\sigma BF(b\bar{b})$	0.8%(2.4%)	7%(21%)	1.3%(4%)	1%(3%)
$\sigma BF(WW^\star)$	–	–	10%(32%)	5%(15%)
$\sigma BF(ZZ^\star)$	–	–	–	62%(190%)
$\Gamma^{\rm tot}_{h_{SM}}$	3%(10%)	25%(78%)	10%(30%)	5%(16%)
Mass (GeV)	**120**	**130**	**140**	**150**
$\sigma BF(b\bar{b})$	1%(3%)	1.5%(5%)	3%(9%)	9%(28%)
$\sigma BF(WW^\star)$	3%(10%)	2.5%(8%)	2.3%(7%)	3%(9%)
$\sigma BF(ZZ^\star)$	16%(50%)	10%(30%)	8%(26%)	11%(34%)
$\Gamma^{\rm tot}_{h_{SM}}$	5%(16%)	6%(18%)	9%(29%)	34%(105%)

- Can now combine with measurements from e^+e^- collisions at NLC and $\gamma\gamma$-collider facility at NLC $\Rightarrow$ determination of fundamental Higgs couplings.

  Example: 4 ways to determine $\Gamma(h \to \mu^+\mu^-)$:

  1) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)BF(h_{SM}\to b\bar{b})]_{\rm MC}}{BF(h_{SM}\to b\bar{b})_{\rm NLC}}$;

  2) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)BF(h_{SM}\to WW^\star)]_{\rm MC}}{BF(h_{SM}\to WW^\star)_{\rm NLC}}$;

  3) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)BF(h_{SM}\to ZZ^\star)]_{\rm MC}\Gamma^{\rm tot}_{h_{SM}}}{\Gamma(h_{SM}\to ZZ^\star)_{\rm NLC}}$;

  4) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)BF(h_{SM}\to WW^\star)\Gamma^{\rm tot}_{h_{SM}}]_{\rm MC}}{\Gamma(h_{SM}\to WW^\star)_{\rm NLC}}$.

  Resulting errors are labelled $(\mu^+\mu^- h_{SM})^2|_{\rm NLC+MC}$ below.

- Use these measurements to distinguish between h_{SM} and h^0 of the MSSM $\Rightarrow$ constraints on H^0 and A^0.

Table 2: Percentage errors (1σ) for combining $L = 600$ fb^{-1} LHC, $L = 200$ fb^{-1}–$\sqrt{s} = 500$ GeV NLC, $L = 50$ fb^{-1} $\gamma\gamma$-collider and MC $R = 0.003\%$ s-channel data, with errors for the latter as quoted in Table 1. Results are presented for two total four-year integrated MC luminosities: $L = 4$ fb^{-1} ($L = 0.4$ fb^{-1}).

Quantity	Errors			
Mass (GeV)	**80**	**100**	**110**	**120**
$(b\bar{b}h_{SM})^2\vert_{\rm NLC+MC}$	6%(10%)	10%(16%)	7%(13%)	7%(13%)
$(c\bar{c}h_{SM})^2\vert_{\rm NLC+MC}$	9%(13%)	12%(18%)	10%(15%)	10%(15%)
$(\mu^+\mu^-h_{SM})^2\vert_{\rm NLC+MC}$	5%(5%)	5%(5%)	4%(5%)	4%(4%)
$(\gamma\gamma h_{SM})^2\vert_{\rm MC}$	15%(18%)	17%(33%)	14%(21%)	14%(20%)
$(\gamma\gamma h_{SM})^2\vert_{\rm NLC+MC}$	9%(10%)	10%(11%)	9%(10%)	9%(10%)
$\Gamma^{\rm tot}_{h_{SM}}\vert_{\rm NLC+MC}$	3%(9%)	8%(16%)	5%(12%)	5%(12%)
Mass (GeV)	**130**	**140**	**150**	**170**
$(b\bar{b}h_{SM})^2\vert_{\rm NLC+MC}$	8%(12%)	9%(10%)	13%(13%)	23%(23%)
$(c\bar{c}h_{SM})^2\vert_{\rm NLC+MC}$	10%(14%)	?		
$(\mu^+\mu^-h_{SM})^2\vert_{\rm NLC+MC}$	4%(5%)	4%	4%(5%)	13%(14%)
$(WW^\star h_{SM})^2\vert_{\rm MC}$	17%(24%)	12%(30%)	33%(104%)	–
$(WW^\star h_{SM})^2\vert_{\rm NLC+MC}$	5%	5%	6%(8%)	10%
$(\gamma\gamma h_{SM})^2\vert_{\rm MC}$	14%(22%)	20%(34%)	48%(110%)	–
$(\gamma\gamma h_{SM})^2\vert_{\rm NLC+MC}$	10%(12%)	13%(15%)	25%(29%)	–
$\Gamma^{\rm tot}_{h_{SM}}\vert_{\rm NLC+MC}$	5%(10%)	6%(8%)	9%(10%)	11%(11%)

– If only s-channel Higgs factory MC data are available ($i.e.$ no Zh NLC or MC data) use

$$(WW^\star h_{SM})^2/(b\bar{b}h_{SM})^2 \propto \sigma BF(WW^\star)/\sigma BF(b\bar{b}).$$

If $110 \lesssim m_{h_{SM}} \lesssim 140$ GeV (a very likely region in the MSSM) then this ratio will be measured with a statistical accuracy of $\lesssim \pm5\%$ for $L_{\rm tot} = 4$ fb^{-1} (see Table 1). Systematic errors of order $\pm5\% - \pm10\%$ from uncertainty in the b quark mass will also enter. A $> 2-3\sigma$ deviation will be observed if $m_{A^0} < 450$ GeV. For $L_{\rm tot} = 0.4$ fb^{-1}, one would observe a $> 1.5-2\sigma$ deviation for

$m_{A^0} < 450$ GeV.

– If $\sqrt{s} = 500$ GeV ($L_{\rm tot} = 200$ fb^{-1}) data available (presumably from NLC operation, but MC could also provide) $\Rightarrow$ use $\Gamma(h \to \mu^+\mu^-)$ (error $\lesssim 5\%$ for either MC $L_{\rm tot} = 0.4$ fb^{-1} or 4 fb^{-1}).

$\Rightarrow$ probes out to $m_{A^0} \gtrsim 600$ GeV for all $m_h \lesssim 2m_W$. No systematics.

– Note that $\Gamma_h^{\rm tot}$ alone cannot be used to distinguish between MSSM and SM in model-independent way. $\Gamma_h^{\rm tot}$ depends on many things, including (in MSSM) the squark-mixing model. However, deviations from SM are generally quite substantial if $m_{A^0} \lesssim 500$ GeV.

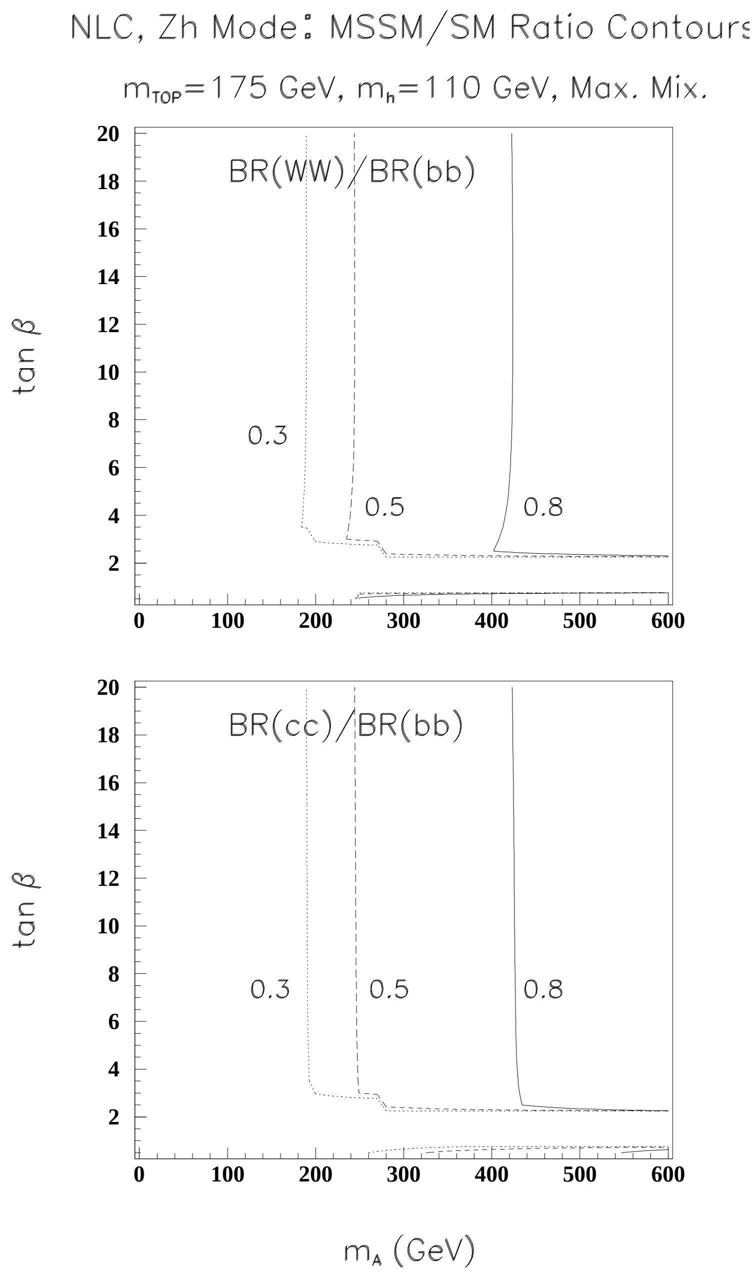

Figure 4: Constant value contours in $(m_{A^0}, \tan\beta)$ parameter space for the rate ratios $[WW^\star/b\overline{b}]_{h^0}/[WW^\star/b\overline{b}]_{h_{SM}}$ and $[c\overline{c}/b\overline{b}]_{h^0}/[c\overline{c}/b\overline{b}]_{h_{SM}}$, for "maximal-mixing" with fixed m_{h^0} = 110 GeV.

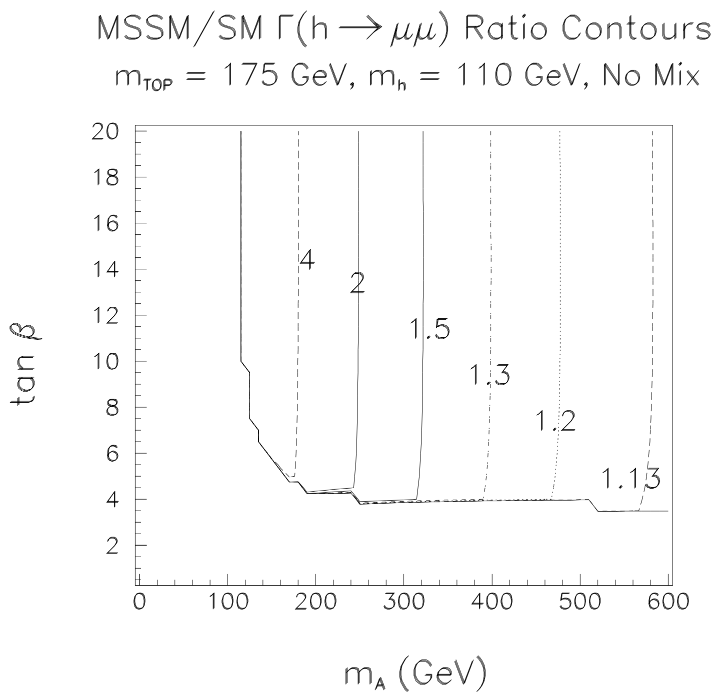

Figure 5: Constant value contours in $(m_{A^0}, \tan\beta)$ parameter space for the ratio $\Gamma(h^0 \rightarrow \mu^+\mu^-)/\Gamma(h_{SM} \rightarrow \mu^+\mu^-)$. We assume "no mixing" in the squark sector and present results for the case of fixed $m_{h^0} = m_{h_{SM}} = 110$ GeV. For "maximal mixing", the vertical contours are essentially identical — only the size of the allowed parameter range is altered. Contours for $\Gamma(h^0 \rightarrow b\bar{b})/\Gamma(h_{SM} \rightarrow b\bar{b})$ are identical.

SUSY H^0 and A^0

- Possibilities for H^0, A^0 are limited at other machines.
  - Discovery of H^0, A^0 is not possible at the LHC for all of $(m_{A^0}, \tan\beta)$ parameter space: $m_{A^0} \gtrsim 200$ GeV, moderate $\tan\beta \gtrsim 3$ is the danger area.
  - At $\sqrt{s} = 500$ GeV, $e^+e^- \to H^0 A^0$ pair production probes only to $m_{A^0} \sim m_{H^0} \lesssim 230 - 240$ GeV.
  - A $\gamma\gamma$ collider could potentially probe up to $m_{A^0} \sim m_{H^0} \sim 0.8\sqrt{s} \sim 400$ GeV with $L \gtrsim 150 - 200$ fb^{-1}.
- $\mu^+\mu^- \to H^0, A^0$ potentially allows production and study of H^0, A^0 up to $m_{A^0} \sim m_{H^0} \lesssim \sqrt{s}$. For $L = 50$ fb^{-1} (5 yrs running at $< \mathcal{L} >= 1 \times 10^{33}$, as possibly achievable for $R \gtrsim 0.14\%$ for $\sqrt{s} = 300 - 500$ GeV):
  - with preknowledge/restrictions on m_{A^0} (from m_{h^0} data, LHC or NLC discovery, *etc.*) $\mu^+\mu^- \to H^0$ and $\mu^+\mu^- \to A^0$ can be studied with precision for all $\tan\beta \gtrsim 1$.
  - without preknowledge, $\mu^+\mu^- \to H^0, A^0$ discovery by scanning from $250 - 500$ GeV at the MC will be possible for most of $(m_{A^0}, \tan\beta)$ parameter space such that they cannot be discovered at the LHC (in particular, if $m_{A^0} \gtrsim 250$ GeV and $\tan\beta \gtrsim 4$).
  - even if the MC is run at $\sqrt{s} = 500$ GeV, the H^0, A^0 can be discovered in the bremstrahlung tail if the $b\bar{b}$ mass resolution (either by direct reconstruction or hard photon recoil) is good enough and $\tan\beta$ is large.

Exotic Higgs Bosons

- If there are doubly charged Higgs bosons, $e^- e^- \to \Delta^{--}$ probes λ_{ee} and $\mu^- \mu^-$ probes $\lambda_{\mu\mu}$ strengths of Majorana-like couplings.

  Current λ limits are such that factory like production of Δ^{--} is possible.

  A Δ^{--} with $m_{\Delta^{--}} \lesssim 500$ GeV will be seen previously at the LHC (for lower masses at TeV33).

  $\mu^- \mu^-$ probes much weaker coupling (e.g. L/R symmetric model strength for see-saw) since narrow Δ^{--} requires $R = 0.01\%$ type beam energy resolution.

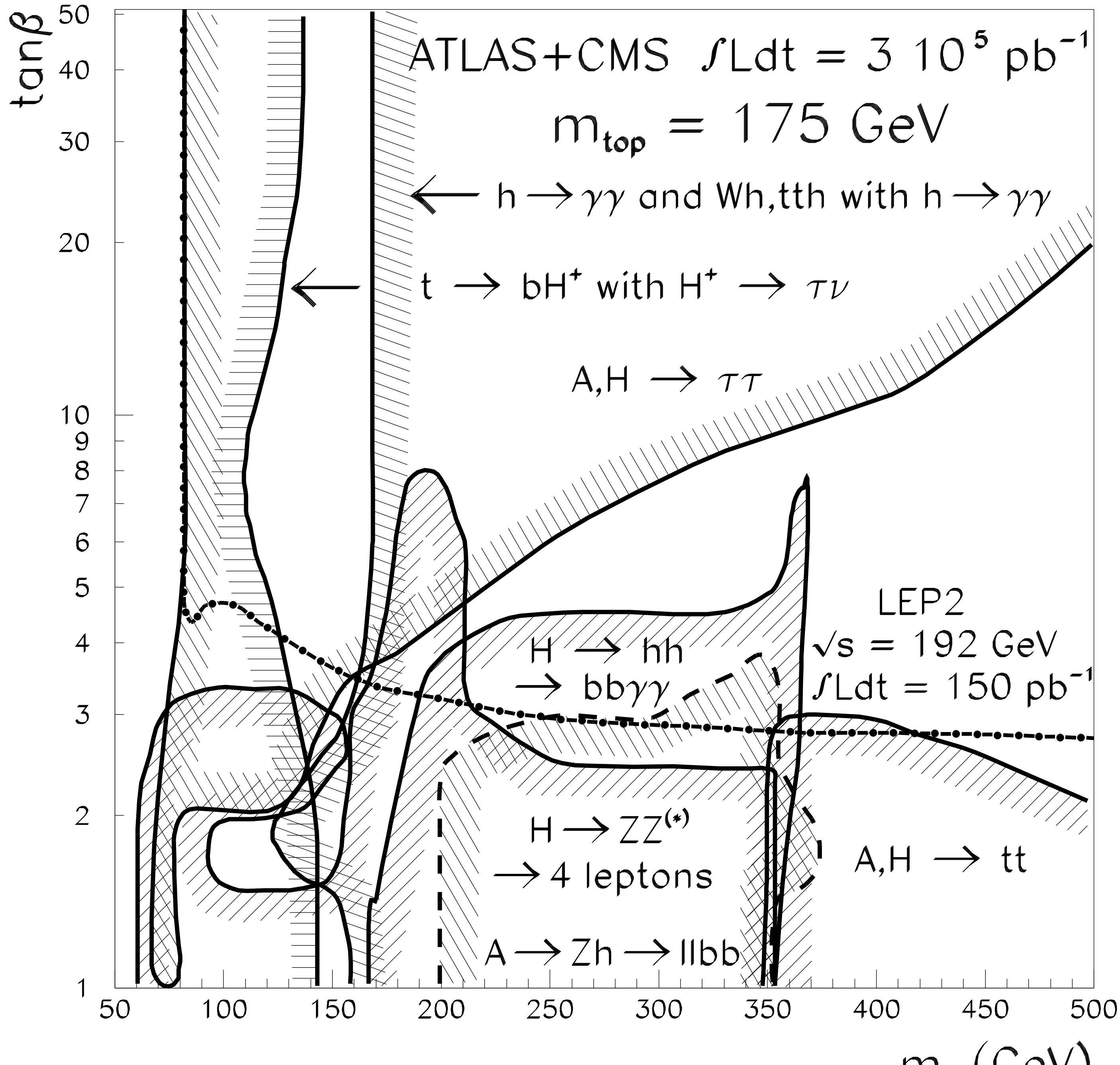

Figure 6: MSSM Higgs discovery contours (5σ) in the parameter space of the minimal supersymmetric model for ATLAS+CMS at the LHC: $L = 300$ fb^{-1} per detector. Two-loop/RGE-improved radiative corrections are included for m_{h^0} and m_{H^0} assuming $m_{\tilde{t}} = 1$ TeV and no squark mixing.

PRECISION MEASUREMENTS OF m_W AND m_t

- At NLC, m_W via $q\bar{q}$ mass reconstruction at $\sqrt{s} = 500$ GeV and m_t via $t\bar{t}$ threshold measurements $\Rightarrow$

$$\Delta m_W = 20 \text{ MeV}, \qquad \Delta m_t = 0.2 \text{ GeV} \qquad (50 \text{ fb}^{-1}, \text{ NLC}) . \quad (0.3)$$

  WW threshold statistics for $L = 50$ fb^{-1} $\Rightarrow$ smaller Δm_W, but systematics from beam energy spread and beam energy uncertainty not adequate to actually make gain. To achieve $L = 50$ fb^{-1} in one year or running the interaction region must be optimized for the given threshold energy. Also it should be noted that at the NLC, errors become systematics dominated for $L > 50$ fb^{-1}; more than one year of optimized running is not useful.

- At MC, WW threshold and $t\bar{t}$ threshold measurements are the most accurate ways to get m_W and m_t. At the muon collider, errors are always statistics dominated given the small R and precise determination of beam energy. The result is that the muon collider can achieve better accuracy for the same integrated luminosity, as displayed in Table 3.

Table 3: Comparison of the achievable precision in m_W and m_t measurements at different future colliders for different $L_{\rm tot}$.

	LEP2		Tevatron		LHC	NLC	$\mu^+\mu^-$		
$L_{\rm tot}$ (fb^{-1})	0.1	2	2	10	10	50	10	50	100
Δm_W (MeV)	144	34	35	20	15	20	20	10	6
Δm_t (GeV)	–	–	4	2	2	0.2	0.2	0.1	0.07

- However, if we assume that $R \sim 0.14\%$ (better is not useful) and use the fixed muon number scaling laws of $\mathcal{L} \propto E^{11/7}$ (assuming an optimized ring is built for the given energy) then we expect $L(\sqrt{s} = 2m_W) \sim 2\ \mathrm{fb}^{-1}/\mathrm{yr}$ and $L(\sqrt{s} = 2m_t) \sim 8\ \mathrm{fb}^{-1}/\mathrm{yr}$, vs. the $L = 50\ \mathrm{fb}^{-1}/\mathrm{yr}$ (either energy) which can be achieved at the NLC *if the interaction region is optimized to the given energies.* In one year of optimized design running, we have the results of Table 4

Table 4: Comparison for the achievable precision in m_W and m_t measurement at NLC vs. MC after 1 year of optimized running.

	NLC	MC
Δm_W (MeV) / L (fb^{-1})	20 / 50	44 / 2
Δm_t (MeV) / L (fb^{-1})	200 / 50	220 / 8

Conclusion: Systematics from beam energy spread *etc.* are low enough at MC that competitive accuracies can be achieved even for conservative yearly luminosities. In allocating running time to $\sqrt{s} \sim 2m_W$ vs. $\sqrt{s} \sim 2m_t$, one should keep in mind:

– what is happening (or has happened) at other machines;

– the fact that $L > 50\ \mathrm{fb}^{-1}$ is not useful at the NLC; and

– the fact that precision electroweak tests are optimized for $\Delta m_W/\Delta m_t \sim 0.02$.

The above table suggests that a fruitful division of labor might be for NLC to focus on $\sqrt{s} \sim 2m_W$ and MC to focus on $\sqrt{s} \sim 2m_t$.

LEPTOQUARKS AND/OR CONTACT INTERACTIONS

- Accept HERA data of excess in e^+q mass distribution at high Q^2 as evidence of a leptoquark (LQ) or new contact interaction.

- $\Rightarrow$ mandatory to search for analogous phenomenon in μq channels at a μp collider, a natural for a muon collider beam (note both signs available) colliding with protons from driver.

- Want $\mu^+\mu^-$ as well as e^+e^- collisions to probe the phenomenon in $\mu^+\mu^-, e^+e^- \to q\overline{q}$ cross channel

Leptoquark example: Compare ep collisions at HERA ($\sqrt{s} \sim 314$ GeV) to μp collisions of the Higgs-factory 50 GeV muon beams with the 1 TeV Fermilab Tevatron beam at the Main Injector ($\sqrt{s} \sim 447$ GeV).

- Extract μ beam with $R \sim 0.1\%$ (*i.e.* before compression to $R = 0.003\%$ or with compression turned off) $\Rightarrow L \sim 1$ fb^{-1}/yr vs. $L \sim 0.1$ fb^{-1}/yr at HERA.

- At HERA, probably the 'observed' $LQ = ed$ or eu. To avoid FCNC the μ-type leptoquarks are most naturally the μs and μc analogues.

- Denote the $LQ \to \ell q$ coupling as $\lambda^J_{\ell q}$, where $J =$ spin of the LQ.

- Take $M_{LQ} = 200$ GeV and $BF(LQ \to \ell q) = 1$ for all.

- Results:
  - 5 LQ events are predicted at HERA with $L = 0.1$ fb^{-1} for:
    $\lambda^0_{eu} = 0.006$, $\lambda^0_{ed} = 0.012$, $\lambda^1_{eu} = 0.004$, $\lambda^1_{ed} = 0.008$.
    (The observed HERA excess $\Rightarrow \lambda^0_{e^+d} \sim 0.025$.)
  - At this same $M_{LQ} = 200$ GeV, the Higgs-factory/MI μp collider with $L = 1$ fb^{-1} yields 5 LQ events for:
    $\lambda^0_{\mu c} = 0.007$, $\lambda^0_{\mu s} = 0.006$, $\lambda^1_{\mu c} = 0.005$, $\lambda^1_{\mu s} = 0.004$.

Conclusion: Since $\lambda_{\mu q}$ values probed at Higgs/MI μp collider are similar or smaller than λ_{eq} values probed at HERA, and since 2nd family leptoquark couplings will probably be larger than 1st family couplings, the Higgs-factory/MI μp collider would be a very important facility if leptoquarks exist.

R-PARITY VIOLATING SCENARIOS

Suppose there is $\not{R}$ of form (baryon number ok, but lepton number is violated):

$$\lambda_{ijk}\widehat{L}_L^i\widehat{L}_L^j\widehat{\overline{E}}_R^k+\lambda'_{ijk}\widehat{L}_L^i\widehat{Q}_L^j\widehat{\overline{D}}_R^k$$

- $\lambda' \neq 0 \Rightarrow$ LQ=squark interpretation of HERA events: most likely $e^+d \to \tilde{t}$ or $\tilde{c}$.

  Once again, crucial to study muon analogues

- $\lambda \neq 0 \Rightarrow$ possibility of $e^+e^- \to \tilde{\nu}_\tau$ (λ_{131}) and/or $\tilde{\nu}_\mu$ (λ_{121}) and $\mu^+\mu^- \to \tilde{\nu}_\tau$ (λ_{232}) and/or $\tilde{\nu}_e$ (λ_{122}).

  Some details:

  - Lightest $\tilde{\nu}$ probably $\tilde{\nu}_\tau$, for which limits on λ_{131} and λ_{232} are relatively weak, $\lambda \lesssim 0.1$.
  - $\lambda_{121} \lesssim 0.04$ and $\lambda_{212} \lesssim 0.04$ limits are not much stronger.
  - Scenarios for decay are:
    * $\tilde{\nu}_\ell \to \nu_\ell\widetilde{\chi}_1^0$ if $m_{\widetilde{\chi}_1^0} < m_{\tilde{\nu}}$ (with $\widetilde{\chi}_1^0$ in turn decaying via virtual $\tilde{\nu}$'s);
    * $\tilde{\nu}_\ell \to \ell^{+\prime}\ell^{-\prime}$ (if $\lambda' \ll \lambda$);
    * $\tilde{\nu}_\ell \to q\overline{q}$ (if $\lambda \ll \lambda'$).
  - Cross section depends on $\Gamma_{\tilde{\nu}}^{\rm tot}$ vs. $\sigma_{\sqrt{s}}$ (cf. Higgs formula): $\Rightarrow$ substantial sensitivity, especially if $\Gamma_{\tilde{\nu}}^{\rm tot}$ is small and $\sigma_{\sqrt{s}}$ is comparable or smaller.
  - In most likely case, LHC and/or normal NLC data show $\not{R}$ and $\Rightarrow$ some knowledge of $m_{\tilde{\nu}}$.

    But, *actual magnitude of λ and λ' will not be easily extracted.*

– MC technique depends upon R and $\Gamma_{\tilde{\nu}}^{\rm tot}$.

– One example:

$\lambda'=0$, $\lambda=0.1$, $m_{\tilde{\nu}}=100$ GeV, $m_{\widetilde{\chi}_1^0}=90$ GeV ($\tilde{\nu}\to\nu\widetilde{\chi}_1^0$ decay allowed), $\Gamma_{\tilde{\nu}}^{\rm tot}=0.051$ GeV.

* For this case, $R=0.003\%\Rightarrow$ best results — correspondingly assume $L=0.1~{\rm fb}^{-1}/{\rm yr}$, reduced by $\epsilon=0.5$ efficiency factor.
* $\Rightarrow$ $\tilde{\nu}$ is observable at $S/\sqrt{B}=3$ level if accumulate $L=0.0008~{\rm fb}^{-1}$ at $\sqrt{s}=m_{\tilde{\nu}}$. To scan 1 GeV interval (expected NLC uncertainty) requires ~ 20 settings, or $L=0.0168~{\rm fb}^{-1}$ (a fraction of a year).
* Then, center and scan $\tilde{\nu}$ peak with $L=0.1~{\rm fb}^{-1}$,

  $\Rightarrow$ about $L=0.05~{\rm fb}^{-1}$ equivalent on-peak measurement of $\mu^+\mu^-\to\tilde{\nu}\to\mu^+\mu^-$ rate $\propto [BF(\tilde{\nu}\to\mu^+\mu^-)]^2$ — $S=1.23\times10^6$, $B=1.28\times10^3\to$ accuracy systematics dominated, lets say $\pm5\%$.

  $\Rightarrow$ very accurate determination of $\Gamma_{\tilde{\nu}}^{\rm tot}$ since $\sigma_{\sqrt{s}}\sim 0.002$ GeV $\ll \Gamma_{\tilde{\nu}}^{\rm tot}$ — lets say $\pm5\%$ (must run program still).
* $\lambda^4\propto[\Gamma_{\tilde{\nu}}^{\rm tot}]^2[BF(\tilde{\nu}\to\mu^+\mu^-)]^2$ determination to $\lesssim\pm10\%$, corresponding to λ determination to $\lesssim\pm2.5\%$!!

– Much worse $\sigma_{\sqrt{s}}$ ($R\sim3\%$ effectively) at e^+e^- collider implies really need the much higher luminosity possible for interaction region especially designed for high $\mathcal{L}$ at low $\sqrt{s}$.

STRONG WW INTERACTIONS

- If no light SM-like Higgs is found at LHC, NLC or MC, then WW sector will be strongly interacting.
- To fully explore, need energy: $\sqrt{s} \sim 3-4$ TeV, with appropriately matched luminosity, is very much better than $\sqrt{s} \sim 1.5 - 2$ TeV. At $\sqrt{s} \sim 3 - 4$ TeV, expected MC luminosities imply that statistics will be such that:
  - * We can study all isospin channels, $I = 0, 1, 2$ *as a function of WW mass* $\Rightarrow$ close analogy to $\pi\pi$ scattering — $\mu^+\mu^+$ collisions are needed as well as $\mu^+\mu^-$.
  - * We can separate $W_L W_L$, $W_L W_T$ and $W_T W_T$ polarization channels from one another and, thereby, clearly separate new physics in these different channels.

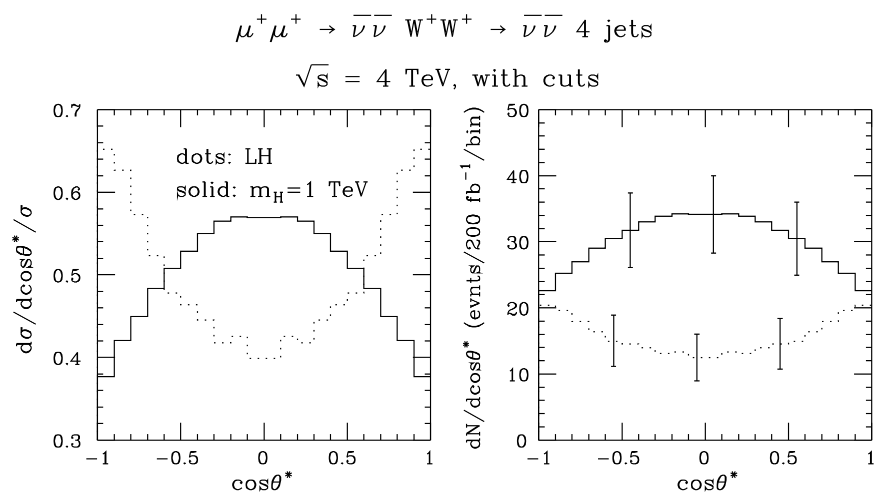

Figure 7: Plots of normalized cross section shapes and $dN/d\cos\theta^*$ (for $L = 200$ fb^{-1}) as a function of the $\cos\theta^*$ of the W^+ decays in the W^+W^+ final state. Error bars for a typical $dN/d\cos\theta^*$ bin are displayed. For these two plots we require $M_{VV} \geq 500$ GeV, $p_T^V \geq 150$ GeV, $|\cos\theta_W^{\rm lab}| < 0.8$ and $30 \leq p_T^{VV} \leq 300$ GeV.

$\Rightarrow$ ability to lay out in detail the effective 'chiral' $\mathcal{L}$ for the WW sector.

STANDARD SUSY STUDIES

– How heavy are SUSY particles?

  * Although fine-tuning considerations suggest that the lightest gauginos should have $m_{\widetilde\chi} \lesssim 200-400$ GeV, sfermions, especially the $\tilde u$ and $\tilde d$ squarks, can have masses $\gtrsim 1$ TeV without violating either fine-tuning or naturalness/hierarchy.
  * Gauge unification 'best' (*i.e.* lower α_s at scale m_Z is possible) if there are SUSY particles above 1 TeV.

– LHC will probably allow 'detection' of very heavy SUSY particles, but will not allowed detailed study of their properties (backgrounds + low event rates).

– $\Rightarrow$ for heavy sparticles, higher energy reach of muon collider would be crucial, especially for β^3 p-wave scalar pair production, for sorting out and study of their properties.

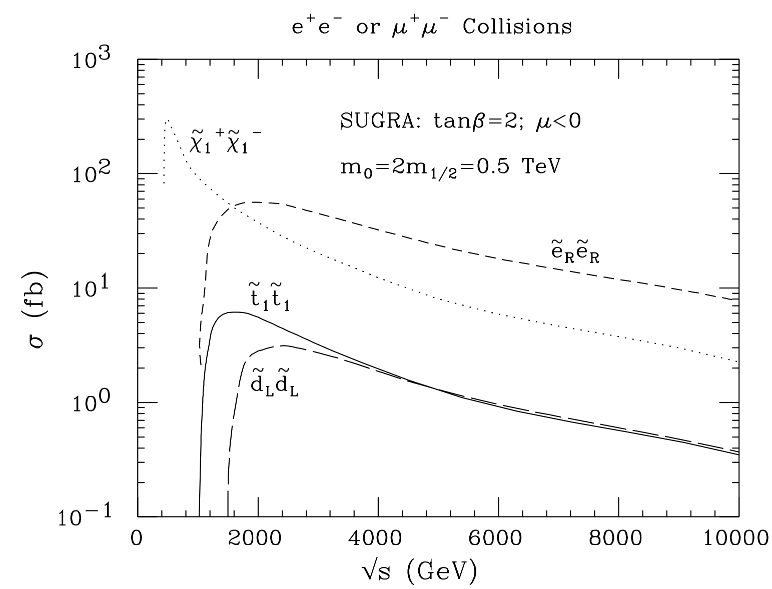

Figure 8: The production cross sections for SUSY particles in a supergravity model with heavy scalars.

NEW Z'

– Easily discovered in the bremstrahlung tail when running the muon collider at high $\sqrt{s}$.

– Reset $\sqrt{s}$ and build cheap new storage ring for a Z' factory.

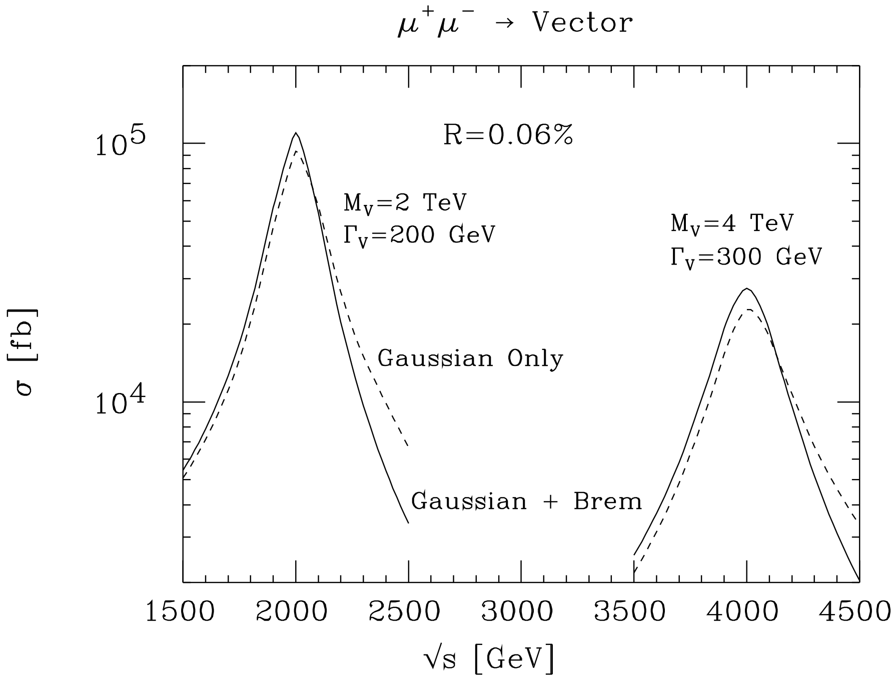

Figure 9: High event rates are possible if the muon collider energy is set equal to the vector resonance (Z' or $\rho_{\rm TC}$) mass. Two examples are shown here with $R = 0.06\%$.

CONCLUSIONS

There will be an enormous number of very important physics studies that should be performed at the next generation of machines.

$\Rightarrow$ We will need many machines in order to have enough luminosity to complete these studies in a timely fashion.

A muon collider would make major contributions to any foreseeable new physics study, and could prove of special value in studying the Higgs sector and the lepton flavor dependence of many other types of new physics.

The physics motivations for a MC are undeniable. We should proceed with the R&D required to assess viability, at the most rapid possible 'natural' pace.